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Market Power and the Power Market: Multiunit Bidding and (In)Efficiency of the Italian Electricity Market*

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Abstract

In this paper we apply auction theory to the modelling of competition in a multi-unit wholesale electricity market. Bidding strategies are examined under different assumptions about the underlying market structure. Results indicate that the application of a competitive pricing rule incentivates bid shading whose extent is affected by the different endowments of generation capacity of multi-plant firms. The inefficiency of the resulting allocation is also discussed.

Policy indications for the Italian case are contained in a final section.

1 Introduction

Electricity markets, once strongly characterized by direct government intervention - motivated by market failure arguments and frequently implemented by vertically integrated public enterprises - are being subjected to radical transformation throughout the world. Reforms have introduced new institutional frameworks intended to easy competitive entry, to provide incentives to efficiency in the generation, transmission, distribution and retailing of output, to reduce tariff and permit direct access. This latter aspect of the new wave of regulatory policies has strong implications in terms of eliminating the need to regulate final consumer price. Instead of regulating prices, the government may regulate the access to the wire system by designing a mechanism for an efficient price-quantity exchange at the wholesale level without granting any monopoly franchise. Starting with the example provided by the England & Wales electricity market regulation introduced

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in the UK in 1990 (Bowen and Dunn, 2001: 567; Green, 1998), in many countries such a mechanism has taken the form of a pool-based market for physical supply of bulk electricity[1]. The basic idea is that an auction for power supply can induce firms to "play a game" in determining who will be producing how much and against which price (Brunekreeft, 2001: 100). How this game can be appropriately modelled by current economic theory is not, however, an entirely settled question. Two different theoretical frameworks have emerged in the last years to nest the modelling of the quantity-price exchange in the electricity market. One approach follows the supply function equilibrium model and examines the characteristics of a spot market equilibrium under demand uncertainty. The other approach analyses the pool market competition using tools of auction theory. For the reasons that will be discussed later, this paper follows this latter approach. In particular, we analyse the outcomes of auctioning power supply under the assumption of cost and demand uncertainty and by supposing that bidders - as in real world situations - will be multi-plants generators characterized by cost differentials. The paper shows that under reasonable assumptions about technology the System Marginal Price rule (SMP) leads to insincere bidding behaviour (bid shading) on the part of generators and that this bid shading increases with bidder's dimension.

The paper is organized as follows. In section 2 we present a short review of the two theoretical approaches to the modelling of competition in the electricity market followed in the literature. In this connection we try and justify the choice of the approach based on the application of auction theory. In section 3 we discuss the general assumptions about cost, demand and auction rules that will be followed in the rest of the paper. Special attention will be devoted to the explanation of what is meant by SMP. In section 4 we analyse the operating of a power market characterized by the presence of a large firm competing against smallest firms operating as a fringe. In section 5 we consider a different market structure characterized by the presence of two large firm interacting strategically. Section 6 contains the analysis of some data on prices and degree of market power obtained from a simulation of the Italian power market as it would be operating on the basis of a full information benchmark. Conclusions are contained in a final section.

2 Different theoretical approaches to the modelling of competition in the electricity market

Two different theoretical approaches have been followed in the last years to model the quantity-price exchange in the electricity market.

[1] For a discussion of the recent liberalization policies followed in various countries the reader is referred to Pollitt (1997).

One approach describes market functioning in terms of the so-called supply function equilibrium model first proposed by Klemperer and Meyer (1989). In this vein Bolle (1992), Green and Newbery (1992), Green (1996) and Newbery (1998) have modelled the energy exchange by examining equilibrium in a spot market with no capacity constraints for generators and no contracts. Newbery (1998) extends this to the case of contracts. Common to these papers is the idea that the only source of uncertainty stays on the demand side of the market. On the contrary, costs of generation are assumed to be known to all market participants. Green and Newbery (1992: 933) claim that the electricity spot market is probably the best example of a market characterized by a supply function equilibrium (SFE) and consider the case of a symmetric duopoly/oligopoly. They derive an equilibrium in supply schedule which is considerably above marginal operating costs. Theoretical results appear to be confirmed by the empirical simulations of the British spot market which shows that market power can be exercised without collusion.

In the SFE approach the net demand facing firm i is always thought to be a residual demand given by the difference between total demand minus the supply made by the opponent. This means that it is always assumed that there is no competition for the merit order and that both duopolists assume they have to serve residual demand. The absence of any reference to the competition for a favorable position in the merit order seems to contradict the very nature of an electricity auction where the position gained in the merit order of bids matters in terms of expected profits and hence in terms of optimal strategy. This is true in particular when multi-plant generators are considered, a case which has not been dealt with so far in the SFE approach.

The alternative approach describes the market outcomes making use of the theoretical tools of multi-unit IPV auction theory where a sealed bid auction is postulated and a competitive price rule is employed to obtain a unique System Marginal Price (SMP) to be payed to all dispatched units irrespective of their bids. This approach was first proposed in a paper of von der Fehr and Harbord (1993) who consider independent generators which can be ranked on the basis of their (constant) marginal production costs. The model is able to capture the essence of an electricity auction where bidders, given the variability of demand and the merit order, have a different probability of setting the SMP. Von der Fehr and Harbord (1993) show that there exist pure strategy equilibria only for the case of low-demand, that is when (fixed) demand can be covered by just one operator who wins at a price equal to the cost of the (next) less efficient operator. In this case it is shown that the equilibrium results from competition among generators for being dispatched. On the contrary, during periods of high demand it is shown (Proposition 3) that SMP converges to the highest admissible price. According to von der Fehr and Harbord (1993: Proposition 5), under demand variability players strike a balance between a high bid, which results in a higher SMP in the event that the bidder becomes marginal, and a low bid which increases the probability of being dispatched.

The assumption of cost variability among bidders is taken up by Wolfram (1998) who derives a bid shading function and presents empirical results obtained from the England and Wales electricity pool. The evidence suggests that the companies increase their bids to raise the price they are paid for inframarginal capacity. In particular generators bid larger markups for units with high marginal costs, i.e. those that are likely to be used after a number of other units are already operating. The latter finding appears to be more pronounced for the larger competitor. Last, there is some evidence that bids for a given unit are higher when more of the units likely to run before that unit are available to supply electricity.

Brunekreft (2001) examines the electricity pool as a multi-unit auction and allows explicitly for multiple-plant generators, an assumption already made by von der Fehr and Harbord (1993) but not fully exploited in their model. He formally derives a bidding rule which extends Proposition 2 of von der Fehr and Harbord (1993) for the multiple-plant context. Firms use a weighted average of the marginal cost and the bid of the next expensive unit as the reference for the lower bidding rule.

Our paper follows the auction approach stream of von der Fehr and Harbord (1993) and Brunekreft (2001). However we set the problem by assuming cost uncertainty and differences in capacities among generators. We derive bidding strategies in two basic contexts, namely i) a large competitor vs. a fringe of small generators, and ii) an oligopolistic setting. In both cases demand will be considered either fixed or variable but always price-insensitive as in von der Fehr and Harbord (1993).

We believe that this approach is useful for modeling competition in the newly established Italian wholesale electricity market which will be activated in the next months. This is so because case i) above will reflect market conditions that will initially prevail when the auction procedure will start. Then when the privatization process will be completed by means of the selling of the three Gencos disentangled from the current quasi-monopolist, the market characteristics will resemble more case ii).

3 General assumptions

The first block of assumptions is about producers and costs.

Assume there are N firms in the market. Each firm, except when differently specified, operates on two types of plants, baseload and peak-load. Call $\left[0, \bar{q}_{\alpha}^{i}\right]$ and $\left[0, \bar{q}_{h}^{i}\right]$ the capacity range for each $i \in N$ and for the two types of plants.

We assume that each bidder i, $i \in N$, observes his private costs, which we call $c_{\alpha}^{i}, c_{h}^{i}$ for base and peak-load plants respectively. For each bidder

i, $i \in N$, opponents' costs $\mathbf{c}^{-i} = [c_1, ..., c_{i-1}, c_{i+1}, ...c_N]$, will be symmetrically assumed to be a random variable drawn from a continuous distribution function $F^{-i}(\mathbf{c})$ with $f^{-i}(.) = F'(.)$ over a support $[(\bar{c}_\alpha, \bar{c}_\alpha), (\bar{c}_h, \bar{c}_h)]$, with $\bar{c}_\alpha \leq \bar{c}_h$. Costs are *i.i.d.*. As a result the auction will be investigated under the hypothesis that it is an IPV auction.

Since in the literature on auctions for electricity cost uncertainty is not a standard assumption it needs justifications. We believe that there is at least one cost component which varies among plants and that other bidders can assume to be random. This component is the opportunity cost associated to availability of a competitors' plant on a given unit period. When side contracting is allowed (i.e. the buying and selling of electricity outside the auction) a plant may not be available for dispatching in the auction. As stressed by Wolfram (1998, 709) if a plant is unavailable a bidders's cost function shifts up. As a result, a plant cost depends upon availability and this information is not of public knowledge. Moreover, one may observe that the hypothesis of cost uncertainty seems to be a logical implication of the hypothesis of multi-plant production. Finally, we assume that each producer is assumed to be a risk neutral profit maximizer.

The second set of assumptions is about the auction rules.

We postulate that the rule of the auction is that market price is equal to the last accepted bid (System Marginal Price, SMP, rule), given demand. Each bidder submits two bids, b^i_α and b^i_h for base and peak capacity, constructed as a correspondences between quantity produced and price.

The auctioneer calculates the merit order of all bids and then derives the aggregate supply function first despatching - up to their capacities - the plants for which the lowest bids have been submitted. The equilibrium price corresponds to the bid made by the marginal bidder (on the base of a total merit ordering of bids). This price is payed to all dispatched suppliers-plants, irrespective of the bids made by the non marginal producers. More formally, call

$$\left(b_{(1)}, ..., b_{(2N)}\right)$$

the sequence of ordered bids and

$$\left(q^{b_{(1)}}, ..., q^{b_{(2N)}}\right)$$

the (not ordered) quantity values associated to each bid.

Call Q the total demand. The SMP $b_{(m)}$ is implicitly derived from:

$$Q - \sum_{i=1}^{(m-1)} \bar{q}^{b_{(i)}} = q^{b_{(m)}}$$

where m indicates the marginal firm. Bids below $b_{(m)}$ can be called infra-marginal and bids above it extra-marginal.

In the paper demand will be assumed to be either fixed and announced before the auction or variable on the basis of some price insensitive random variable. This ways of modeling the demand side of the market are similar to the hypotheses considered by von der Fehr and Harbord (1993) and by Brunekreeft (2001).

At the end of this multi-unit competitive auction three possible outcomes can emerge: a producer is simply dispatched (and becomes an infra-marginal producer) and receives the price proposed by the marginal bidder for the whole (base and peak) quantity he supplies; a producer is both dispatched and fixes the price (marginal producer) and receives his price for the whole (base and peak) quantity he supplies; a producer is not dispatched for the peak capacity and is paid the SMP for baseload capacity only. No cost for the pure participation to the auction is assumed to exist as well as no remuneration for the pure demonstration of a willingness to participate.

We turn now to the definition of the efficiency of the mechanism. In general, a multi-unit procurement auction produces an efficient allocation when the sellers are called in on the basis of a merit order of bids which is a one-to-one mapping of the merit order of the true production costs. In practice this implies, in our case, that electricity should be generated by the lowest costs plants. When cost are assumed to be constant, Ausubel and Cramton (1998: *Lemma 1*) show that the efficiency of a multi-unit IPV auction requires symmetric and flat bid functions so that for almost every c: $B^i(q(c), c) = b(c)$ for $q \in [0, \bar{q}]$ with $b(.) : [\bar{c}_h, \bar{c}_h] \rightarrow R_+$ non decreasing almost everywhere for every bidder. In our case this caracterisation of the equilibrium must be extended in order to take the multi-unit assumption into account.

On the basis of these general assumptions in the following section we analyse the result of the auction postulating alternative configurations of the industry structure.

4 Many atomistic bidders vs. one large bidder when demand is fixed

It is very likely that, at the start of his operational life, the Italian electricity market will be characterized by the presence of a very large producer competing against a number of relatively small producers. This justifies the derivation of a possible market equilibrium that in this kind of industry structure the auction may generate. This is the content of this section.

We assume that the set of bidders can be partitioned into two subsets so that:

$$\{S; l\} = \{s_1, ..., s_j, ..., s_{N-1}; l\}$$

with $N = S + 1$. S indicates the set of small producers indexed by j whereas l indicates the large one.

Assume that small producers own a single plant having constant marginal costs $c_h^j, \forall j \in S$. Call c_α^l, c_h^l the marginal cost of production of the large producer who is assumed to own two plants, one base-load and one peak-load .

Demand is fixed in the sense that the auctioneer announces a load Q to be allocated by the bidding process. This volume is known to all participants when they submit bids. In a sense this makes the auction similar to a uniform price procurement auction with multiple units and allocations.

Consider the base-load plants first. Given the general assumptions made about costs in section 3 we can say that whenever the most efficient peak plant is dispatched, all the base plants will be producing with probability one. This permits us to concentrate on the bidding strategy for peak capacity only[2]. Let Q_α indicate the baseload production and let $\bar{Q} = Q - Q_\alpha$ indicates the total peak quantity and define the market equilibrium as

$$\bar{Q} - \sum_{i=1}^{(m-1)} \bar{q}_h^{b_{(i)}} \leq \bar{q}_h^{b_{(m)}}$$

where $b_{(m)} \equiv SMP = \sup \left\{ b \mid \bar{Q} - \sum_{i=1}^{(m-1)} \bar{q}_h^{b_{(i)}} \leq \bar{q}_h^{b_{(m)}} \leq \bar{q}_h^{b_{(m+1)}} \right\}$ and $(m) \in [(1), ..., (N)]$ is the position in the ordered statistics of bids of the last accepted bid.

Since the base-load plants are dispatched with probability one and they have no chance of being the marginal producer, they can do no better than bid at cost.

As for peak plants, the bidding behaviour of generators is very likely to depend upon the chace they have to be the marginal producer. When a firm sets the market price and she is a multi-unit seller, she has the incentive to increase the bid and to earn larger profits on baseload production. In the following we concentrate on the bidding behaviour of the large producer assuming that each small producer j follows a truth telling strategy and bids at her cost, i.e. that for each j $b_h^j(.) = c_h^j$. When small producers bid truthfully what the best profit maximizing reply of l might be? To find this reply we start by defining the expected profit of the large producer. Her expected profit has 3 components:

$$\begin{aligned} E\left[\Pi(\mathbf{c}, b_h^l)\right] &= \Pi^1(\mathbf{c}, b_h^l) \Pr[\text{being } \textit{one} \text{ of the infra-marginal producers}] + \\ &\quad \Pi^2(\mathbf{c}, b_h^l) \Pr[\text{being } \textit{the} \text{ marginal producer }] + \\ &\quad \Pi^3(\mathbf{c}, b_h^l) \Pr[\text{being } \textit{one} \text{ of the extra-marginal producers}] \end{aligned}$$

The first component corresponds to the case in which, given $b^j(c) = c^j\ \forall j$, l bids $b_h^l(c) < b^{(m)}(c)$ and she is dispatched for both base and peak quantities

[2]When demand is known to bidders only the residual demand (i.e. total demand minus baseload demand) matters.

at the maximum capacity. Consequently, she receives the SMP fixed by somebodies' else bid which is higher than her own bid.

The second part of the expected profit corresponds to the situation in which l determines the SMP and therefore receives her bid for both base (at the maximum load capacity) and peak (not necessarily at the maximum capacity) quantities.

The third part of the expected profit represents the case in which l submits $b_h^l(c) > b^{(m)}(c)$ and therefore she is not dispatched for peak quantity and only receives SMP for her base capacity.

Define $F_{(.)}(c)$ as the distribution of the order statistics of generation costs and $F_{(.)}^{-l}(c)$ the subjective distribution of l about the opponents' costs. Assume that the market administrator announces a fixed quantity Q. To simplify notations and without loss of generality assume that $\bar{q}_h^j = \bar{q}_h, \forall j \in S$, so that $q_h^l = \bar{Q} - (m-1)\,\bar{q}_h \leq \bar{q}_h^l$ is the residual quantity to be produced when the large generator becomes marginal. When demand is fixed, m is known since capacity levels are not assumed to be random.

Considering all the components of the expected profit of l we have:

$$E\left[\Pi(.)\right] = \bar{q}_h^l \int_{b_h^l}^{\bar{c}^h} \left(\hat{b} - c_h^l\right) d\left[F_{(m-1)}^{-l}(\hat{b})\right] + \bar{q}_\alpha^l \int_{b_h^l}^{\bar{c}^h} \left(\hat{b} - c_\alpha^l\right) d\left[F_{(m-1)}^{-l}(\hat{b})\right] +$$
$$\left[q_h^l\left(b_h^l - c_h^l\right) + \bar{q}_\alpha^l\left(b_h^l - c_\alpha^l\right)\right]\left[F_{(m-1)}^{-l}(b_h^l) - F_{(m)}^{-l}(b_h^l)\right] +$$
$$\bar{q}_\alpha^l \int_{\bar{c}^h}^{b_h^l} \left(\hat{b} - c_\alpha^l\right) d\left[F_{(m)}^{-l}(\hat{b})\right] \tag{1}$$

Differentiation w.r.t. b_h^l yields after equating to zero and rearranging:

$$b_h^l = c_h^l + \frac{\left(\bar{q}_\alpha^l + q_h^l\right)\left[F_{(m-1)}^{-l}(b_h^l) - F_{(m)}^{-l}(b_h^l)\right]}{\left(\bar{q}_h^l - q_h^l\right) f_{(m-1)}^{-l}(b_h^l) + q_h^l f_{(m)}^{-l}(b_h^l)} \tag{2}$$

Equation (2) implies a bid higher than marginal costs of generation. Although (2) does not define a proper bid function, yet it is easy to verify that the amount of bid shading increases with the quantity supplied by the base-load plant. This accords with previous results found in the literature and with the intuition stemming from the auction rules. Wolfram (1998, 710), for instance, obtains a similar result in the context of different hypothesis and she stresses that the incentive to manipulate the bid is increasing in the number of inframarginal units for which the marginal bid is likely to set the price. On the contrary, we cannot provide general conditions under which the implicit bid function is monotonically increasing (or decreasing) with respect to the quantity allocated to the peak-load plant. (2) is increasing with respect to the peak quantity supplied when the following condition hold:

$$\frac{1}{\left(\bar{q}_h^l - q_h^l\right)} - \frac{\left[F_{(m-1)}^{-l}(b_h^l) - F_{(m)}^{-l}(b_h^l)\right] f_{(m)}^{-l}(b_h^l)}{f_{(m-1)}^{-l}(b_h^l) + q_h^l f_{(m)}^{-l}(b_h^l)} < 0$$

so that the behaviour of the function depends upon the level of idle capacity $\left(\bar{q}_h^l - q_h^l\right)$, the number of bidders N and the number of "winners" m.

When $q_h^l \to \bar{q}_h^l$ the limit of (2) is:

$$\begin{aligned} b_h^l &= c_h^l + \frac{\bar{q}_\alpha^l + \bar{q}_h^l}{\bar{q}_h^l} \frac{\left[F_{(m-1)}^{-l}(b_h^l) - F_{(m)}^{-l}(b_h^l)\right]}{f_{(m)}^{-l}} \\ &= c_h^l + \frac{\bar{q}_\alpha^l + \bar{q}_h^l}{\bar{q}_h^l} \frac{\left[1 - F(b_h^l)\right]}{(m-1) f\left(b_h^l\right)} \end{aligned} \tag{3}$$

The latter result means that when both base-load and peak capacity are fully dispatched in equilibrium the markup over cost for the large producer tends to be equal to the expected difference between the cost of the next plant in the merit order and the costs of the large producer given that she occupies the *m.th* position, i.e. provided that she sets the SMP. This is the larger markup that can be applied to costs without incurring in the risk of losing the position in the merit order.

The result that the bid has an upper bound given by the cost of the next firm in the merit order is not novel in the literature. Von der Fehr and Harbord (1993) and Brunekreft (2001) obtain a similar result for the case of common knowledge of costs among producers.

As a final comment we can emphasize the inefficiency of the allocation resulting from a competitive auction mechanism characterised by the presence of at least one multi-unit seller such as the one described above. As discussed in section 3, the efficiency of an auction mechanism where multiple-unit bids are submitted requires flat bid functions for the all range of valuations. This in turn implies, in our case, that for all bidders the amount of bid shading should be constant with respect to the quantity allocated.

We notice that the function (2) is not flat with respect to the peak quantity sold in equilibrium. The only case in which the bid function becomes flat is when the large bidder has a null probability of becoming the marginal producer (in this case she will bid at cost). Since this cannot be assumed in this model, it will be likely that the merit order resulting from the auction will differ from the true merit order of costs and this conclusion holds even in the case in which the large producer sells her full peak capacity in equilibrium (see 3). Moreover, the inefficiency of the allocation is amplified by the presence of the base-load capacity possessed by the dominant firm.

4.1 The case of uncertain demand

We next relax the assumption of fixed demand. When the demand side of the market is assumed to be active, i.e. when it is assumed that buyers are allowed to submit bids for the lots they want to acquire, the assumption of fixed and known demand cannot be maintained and two possibilities should be considered. On the one hand, we may think of buyers submitting bids for lots of energy on the basis of their needs irrespective of the price they would be willing to pay. On the other hand, we may think of buyers who actually base their decisions on the trade-off between price and quantity and therefore submits bids which are negatively sloped demand schedules. In both situations the demand to be served is uncertain to market participants although the source of uncertainty is different in the two cases. In what follows, as in von der Fehr and Harbord (1993), we will only consider the case in which the random component of the demand is independent of price.

When market demand is unknown, generators are invited to submit price-quantity schedules and the cost minimising allocation is determined after the realization of the demand.

We consider a price-inelastic variable demand such that:

$$Q = D\tilde{u}$$

where D is a fixed amount and $\tilde{u}$ is the ralization of a random variable assumed to follow a known continuous distribution $G(u)$ with $g(u) = G'(u)$ over a positive support $[\bar{u},\bar{u}]$. We assume that u and $\mathbf{c}$ are *i.i.d.* As before, we can define the expected profit of the large firm as:

$$\begin{aligned} E\left[\Pi(\mathbf{c}, b_h^l, \tilde{u})\right] &= \Pi^1(\mathbf{c}, b_h^l, \tilde{u}) \Pr[\text{being } \textit{one} \text{ of the infra-marginal producers}] + \\ &\quad \Pi^2(\mathbf{c}, b_h^l, \tilde{u}) \Pr[\text{being } \textit{the} \text{ marginal producer }] + \\ &\quad \Pi^3(\mathbf{c}, b_h^l, \tilde{u}) \Pr[\text{being } \textit{one} \text{ of the extra-marginal producers}] \end{aligned}$$

The joint probabilities of the three possible events are:

$$H^I\left(b_h^l, m\right) = \sum_{m=2}^{n} \left[1 - F_{(m-1)}^{-l}(b_h^l)\right] \left[1 - G\left(\frac{D\underline{u} + \bar{q}_h^l + (m-2)\,\bar{q}_h}{D}\right)\right]$$

$$H^E\left(b_h^l, m\right) = \sum_{m=1}^{n-1} \left[F_{(m)}^{-l}(b_h^l)\right] G\left(\frac{D\underline{u} + m\bar{q}_h}{D}\right)$$

$$H^M\left(b_h^l, m\right) = H^I - H^E$$

where $H^I\left(b_h^l, m\right)$ indicates the probability of being one of the inframarginal producers; $H^E\left(b_h^l, m\right)$ the probability of being one of the extra-marginal producers and finally $H^M\left(b_h^l, m\right)$ is the probability of setting the SMP. Moreover let $h^I\left(b_h^l, m\right)$ and $h^E\left(b_h^l, m\right)$ be the p.d.f. obtained differentiating $H^I\left(b_h^l, m\right)$ and $H^E\left(b_h^l, m\right)$ with respect to the bid.

Under the above assumptions the expected profit can now be written as:

$$E\left[\Pi(u,\mathbf{c})\right] = \bar{q}_h^l \int_{b_h^l}^{\bar{c}^h} \left(\hat{b} - c_h^l\right) d\left[H^I\left(\hat{b}, m\right)\right] + \bar{q}_\alpha^l \int_{b_h^l}^{\bar{c}^h} \left(\hat{b} - c_\alpha^l\right) d\left[H^I\left(\hat{b}, m\right)\right] +$$

$$\left[\tilde{q}_h^l\left(\tilde{u}\right)\left(b_h^l - c_h^l\right) + \bar{q}_\alpha^l\left(b_h^l - c_\alpha^l\right)\right]\left[H^M\left(b_h^l, m\right)\right] +$$

$$\bar{q}_\alpha^l \int_{\bar{c}^h}^{b_h^l} \left(\hat{b} - c_\alpha^l\right) d\left[H^E\left(\hat{b}, m\right)\right] \tag{4}$$

Differentiating (4) with respect to the bid and rearranging the first order conditions we get:

$$b_h^l = c_h^l + \frac{\left(\tilde{q}_h^l\left(\tilde{u}\right) + \bar{q}_\alpha^l\right)\left[H^M\left(b_h^l, m\right)\right]}{\left[\left(\bar{q}_h^l - \tilde{q}_h^l\left(\tilde{u}\right)\right)h^I\left(b_h^l, m\right) + \tilde{q}_h^l\left(\tilde{u}\right)h^E\left(b_h^l, m\right)\right]} \tag{5}$$

It is easy to verify that function (5) is increasing with respect to private costs and that the amount of bid shading is always positive for a positive $\left[H^M\left(b_h^l, m\right)\right]$. In particular, when $q_h^l \to \bar{q}_h^l$ the limit of (5) is:

$$\lim_{q_h^l \to \bar{q}_h^l} b_h^l = c_h^l + \frac{\left(\bar{q}_h^l + \bar{q}_\alpha^l\right)\left[H^M\left(b_h^l, m\right)\right]}{\bar{q}_h^l h^E\left(b_h^l, m\right)}$$

so that the bid shading does not disappear even when she sells her total capacity.

We can therefore conclude that, as in the case of a known demand, sincere bidding requires a null probability of being the price setter. Moreover, the models with fixed and random demand share the same implication in terms of efficiency of the mechanism: in a multi-unit auction when one bidder has some degree of market power the efficiency of the final allocation is not generally obtained.

5 Equilibrium with two bidders and fixed demand

We now consider a market structure in which the supply side is characterized by the presence of two large producers who both may exert some market power. This case has been already analysed by von der Fehr and Harbord (1993) in a context of common knowledge of production costs and single plant producers. In our case let $N = \{1, 2\}$ and let $\bar{q}_\alpha^1, \bar{q}_\alpha^2$ and $\bar{q}_h^1, \bar{q}_h^2$ the generation capacity of the two producers for the two types of plants. Call c_α^1, c_α^2 and c_h^1, c_h^2 the associated marginal costs. As before we assume that each firm observes her own costs whereas the cost of the rival are assumed to be random variables drawn from a common continuous distribution function $F(c)$ with $f(c) = F'(c)$ over a support $[\bar{c}_h, \bar{c}_h]$, with $\bar{c}_h > max\left(c_\alpha^1, c_\alpha^2\right)$. The above *i.i.d.* assumption still applies. Baseload plants are always dispatched first whereas peak plants are known to be called into operation only after the moment in which demand is announced.

We consider a simple model in which the announced market demand is such that only one peak generator is called into operation and hence the two firms compete to be the marginal producer[3]. Therefore, the bidder submitting the highest bid sells her base-load capacity only. The probability of being the marginal producer is equal to the probability of submitting the lowest bid on the peak quantity $(Q - Q_\alpha)$ which is known after Q is announced at the beginning of the auction. In a symmetric equilibrium $b^*(c)$, with $b^*(.)$ representing a bid function monotonically increasing with respect to costs, $b_h^1 < b_h^2$ implies $\sigma\left(b_h^1\right) = c_h^2$, where $\sigma(b) \equiv b^{*-1}\left(\tilde{b}\right)$ indicates the inverse bid function. Hence $[1 - F(\sigma(b))]$ will indicate the probability of 1 being the marginal generator and $f(\sigma(b))\frac{d\sigma(b)}{db}$ is the associated marginal density.

Expected profit for firm 1 is therefore given by the sum of three terms:

$$E\left[\Pi^1\right] = \bar{q}_\alpha^1 \int_{\underline{c}}^{b_h^1} \hat{b} f\left(\sigma\left(\hat{b}\right)\right) \frac{d\sigma\left(\hat{b}\right)}{d\hat{b}} d\hat{b} +$$

$$\bar{q}_\alpha^1 b_h^1 \left[1 - F(\sigma(b))\right] - \bar{q}_\alpha^1 c_\alpha^1 + (Q - Q_\alpha)\left(b_h^1 - c_h^1\right)\left[1 - F(\sigma(b))\right] \qquad (6)$$

The first term represents the expected profit when the other firms wins on the peak unit. Firm 1 then sells baseload capacity only at a price equal to $b_h^2 < b_h^1$. The other two terms represent the expected profit associated to the fact of being the marginal producer. The costs of baseload production are obviously incurred with probability one.

First order conditions for the optimal bid b_h^1 generate:

$$\left(b_h^1 - c_h^1\right) = \frac{\left(Q - \bar{q}_\alpha^2\right)}{(Q - Q_\alpha)} \frac{\left[1 - F\left(c_h^1\right)\right]}{f\left(c_h^1\right)} \frac{db_h^1}{dc_h^1} \qquad (7)$$

Solving (7) for all relevant values of c, and $b^*(\bar{c}) = \bar{c}$, we get:

$$b_h^1 = \frac{\bar{q}_\alpha^1}{\left(\bar{q}_\alpha^1 + q_h^1\right)}\bar{c} + \frac{q_h^1}{\left(\bar{q}_\alpha^1 + q_h^1\right)}\left[c_h^1 + \frac{\int_{c_h^1}^{\bar{c}} \left[1 - F(\tilde{c})\right] d\tilde{c}}{\left[1 - F\left(c_h^1\right)\right]}\right] \qquad (8)$$

[3] This case has been considered by von der Fehr and Harbord (1993) as a low demand case.

We notice from (8) that the optimal bid is a weigthed average of the maximal bid $\bar{c}$ and the bid that would have been optimal had the auction been a discriminative procedure. In fact, if we put $\bar{q}_{\alpha}^{1} = 0$, i.e. if we exclude baseload capacity, the above result reduces to:

$$b\left(c_h^1\right)\big|_{\bar{q}_{\alpha}^1=0} = c_h^1 + \frac{\int_{c_h^1}^{\bar{c}} \left[1 - F\left(\tilde{c}\right)\right] d\tilde{c}}{\left[1 - F\left(c_h^1\right)\right]} \tag{9}$$

which is the standard bid made in a single-unit procurement auction with two bidders.

The result of our simple model implies that when bidders own baseload and peak-load plants they submit a bid on the peak quantity which is higher than the bid they would have submitted in a discriminative auction procedure.

As a final consideration, we notice that, unless $\bar{q}_{\alpha}^{1} = \bar{q}_{\alpha}^{2} = 0$, the bid function is always decreasing with respect to the residual quantity q_h^1 but the bid shading is independent upon the peak capacity $\bar{q}_h^1$. Since the residual quantity is always the same irrespective of the bidder selected, we can conclude that this simple mechanism produces an efficient allocation only when $\bar{q}_{\alpha}^{1} = \bar{q}_{\alpha}^{2}$.

5.1 The case of uncertain demand

We relax again the assumption of fixed demand for electricity and we consider a price-inelastic variable demand $D\tilde{u}$ as before. For each generating unit owned, firms submit a bid function mapping quantity into price up to their capacity. In the following we focus on bidding strategies of firms for peak-plants $h = 1, 2$ since for baseload plants bids are again equal to marginal costs of generation.

When two bidders interact strategically in the peak segment of the market, the degree of demand uncertainty is crucial to the definition of the bidding strategy. On the one hand, we may consider a small range of demand variability so that only one of the two peak plants will be dispatched in equilibrium. Hence the bid will determine a merit order and the generator who presented the smallest bid will cover the residual load, i.e. the load remaining after that all baseload plants have been called into operation. Therefore, strictly speaking, the exogenous uncertainty affects the level of final quantity to be delivered whereas the individual bid determines the identity of the marginal plant[4]. On the other hand, the degree of demand variability may go beyond the capacity range of a single operator[5].

[4] In this respect our model extends von der Fehr and Harbord (1993) who assumed random demand independent of price and common knowledge of costs.

[5] This case parallels the third case of von der Fehr and Harbord (1993) whereas Brunekreeft (2001) only considered a case of demand variability restricted into the capacity range of a single generating unit.

In this second hypothesis, the realization of demand will determine not only the final quantity to be produced but even how many producers will be dispatched (one or both) and who will set the price.

We consider the two cases in turn starting with the hypothesis of low demand variability, we assume that:

$$\begin{aligned} u &\in [\underline{u}, \bar{u}_1] \\ D[\bar{u}_1 - \underline{u}] &\leq \min\left(\bar{q}_h^1, \bar{q}_h^2\right) \\ D\underline{u} &= \bar{q}_\alpha^1 + \bar{q}_\alpha^2 \end{aligned}$$

Each bidder i $(i = (1,2))$ submits a bid function for an ex-ante unknown quantity and produces a realized residual quantity $\tilde{q}_h^1(u) \equiv (D\tilde{u} - D\underline{u})$, provided that he is the lowest bidder. The highest bidder of the two does not produce any peak quantity.

It is easy to show that the equilibrium bid function for the case of low variability of demand is a simple extension of the bid that was found to be optimal in the fixed demand case (see (8)). Optimal bid for firm 1 is given by:

$$\begin{aligned} b^1\left(u, c_h^1\right) &= \frac{\bar{q}_\alpha^1 \bar{c}}{\bar{q}_\alpha^1 + \tilde{q}_h^1(u)} + \\ & \frac{\tilde{q}_h^1(u)}{\bar{q}_\alpha^1 + \tilde{q}_h^1(u)} \left[c_h^1 + \frac{\int_{c_h^1}^{\bar{c}_h} (1 - F(\tilde{c}))\, d\tilde{c}}{\left[1 - F\left(c_h^1\right)\right]}\right] \end{aligned} \tag{10}$$

The bid function (10) maps different realization of $\tilde{q}_h^1(u)$ into a bid price b_h^1, given bidder's 1 costs. The bid function is increasing with respect to private costs c_h^1 and $b_h^1 = \bar{c}$ when $c_h^1 = \bar{c}$. Therefore uncertainty about generation costs and the assumption of common knowledge of cost distribution generate an upper bound to the level of the bid[6].

We now consider the bidding function's behaviour with respect to the quantity that is dispatched after the realization of $\tilde{q}_h^1(u)$. It is easy to verify that:

$$\begin{aligned} b^1\left(c_h^1, \tilde{q}_h^1(u)\right)\Big|_{\tilde{q}_h^1(u)=0} &= \bar{c} \\ b^1\left(c_h^1, \tilde{q}_h^1(u)\right)\Big|_{\tilde{q}_h^1(u)=\bar{q}_h^1} &= \frac{\bar{q}_\alpha^1 \bar{c} + \bar{q}_h^1 \left[c_h^1 + \frac{\int_{c_h^1}^{\bar{c}_h} (1-F(\tilde{c}))d\tilde{c}}{\left[1-F\left(c_h^1\right)\right]}\right]}{\bar{q}_\alpha^1 + \bar{q}_h^1} \end{aligned}$$

and that bid function is decreasing with respect to the realized quantity $\tilde{q}_h^1(u)$. In particular, as the quantity produced by the peak plant increases,

[6] There is no need in this model to introduce a price ceiling or some artificial upper bound in the price level, to prevent an arbitrarily high bid.

the bid function (10) assigns a larger weight to the "first price auction" component of the bid (the term into square brackets).

The latter result allows us to extend to this model the same considerations about the inefficiency of the allocation drawn for the case of fixed demand. The efficiency result is obtained only in the special case in which bidders have equal capacities of both baseload and peak plants and they obtain a total allocation of the capacity offered in the event of being marginal[7]. The presence of an "externality" from peak to baseload production shifts up the bid function for any quantity produced ex-post. An interesting result emerges when baseload quantity $\bar{q}_{\alpha}^{1}$ is set to zero; in this case the bid function coincides with the optimal bid of a discriminative auction model and it is flat with respect to quantity dispatched. Hence, low variability of demand, constant marginal costs, rivals' cost uncertainty and absence of base-load production make the auction mechanism efficient since the bid function is flat. This result accords with intuition if we think that the absence of baseload production and a unique winner in the peak segment of the market make the auction similar to a single object first price auction.

It is interesting to check whether the efficiency result holds also in the case of multiple winners in the peak portion of the market under high variability of demand. In this case we assume that:

$$\begin{aligned} u &\in [\underline{u}, \bar{u}_H], \bar{u}_H > \bar{u}_L \\ D[\bar{u}_H - \underline{u}] &\leq (\bar{q}_h^1 + \bar{q}_h^2) \\ D\underline{u} &= \bar{q}_\alpha^1 + \bar{q}_\alpha^2 \end{aligned}$$

so that demand variation implies that both peak-load plants may be dispatched. Under these hypothesis bidder 1 may turn out to be marginal in two cases:

$$\begin{aligned} i)\ b_h^1 &< b_h^2 \text{ and } D[\tilde{u} - \underline{u}] \leq \bar{q}_h^1 \\ ii)\ b_h^1 &> b_h^2 \text{ and } D[\bar{u}_H - \tilde{u}] \leq \bar{q}_h^1 \end{aligned}$$

Since $\tilde{u}$ and $\tilde{c}$ are assumed to be independent random variables the probability of being marginal - in both events, $i)$ and $ii)$ - is the product of two probabilities. Bidder 1 is inframarginal when:

$$b_h^1 < b_h^2 \text{ and } D[\tilde{u} - \underline{u}] > \bar{q}_h^1$$

and she is extra-marginal when:

$$b_h^1 > b_h^2 \text{ and } D[\bar{u}_H - \tilde{u}] > \bar{q}_h^1$$

In the latter case she sells baseload power only.

[7] The latter consideration is in line with Ausubel and Cramton (1998) result.

Let $G(.)$ be again the distribution of the random variable u, and let $\sigma(.)$ be the inverse bid function.

Expected profit of firm 1, as a function of u, can then be written as:

$$
\begin{aligned}
E\left[\Pi^1(u)\right] &= \left(b_h^1 - c_h^1\right)\tilde{q}_h^1(u)\,G\left(\frac{\bar{q}_h^1 + D\underline{u}}{D}\right)\left[1 - F\left(\sigma\left(b_h^1\right)\right)\right] + \\
&\quad \left(b_h^1 - c_\alpha^1\right)\bar{q}_\alpha^1 G\left(\frac{\bar{q}_h^1 + D\underline{u}}{D}\right)\left[1 - F\left(\sigma\left(b_h^1\right)\right)\right] + \\
&\quad \left(b_h^1 - c_h^1\right)\tilde{q}_h^1(u)\left[1 - G\left(\frac{D\bar{u}_H - \bar{q}_h^1}{D}\right)\right]\left[F\left(\sigma\left(b_h^1\right)\right)\right] + \\
&\quad \left(b_h^1 - c_\alpha^1\right)\bar{q}_\alpha^1\left[1 - G\left(\frac{D\bar{u}_H - \bar{q}_h^1}{D}\right)\right]\left[F\left(\sigma\left(b_h^1\right)\right)\right] + \\
&\quad \left\{\bar{q}_h^1 \int_{b_h^1}^{\bar{c}_h}\left(\tilde{b} - c_h^1\right)\left[f\left(\sigma\left(\tilde{b}\right)\right)\right]\frac{d\sigma}{d\tilde{b}}d\tilde{b} + \right. \\
&\quad \left.\bar{q}_\alpha^1 \int_{b_h^1}^{\bar{c}_h}\left(\tilde{b} - c_\alpha^1\right)\left[f\left(\sigma\left(\tilde{b}\right)\right)\right]\frac{d\sigma}{d\tilde{b}}d\tilde{b}\right\}\left[1 - G\left(\frac{\bar{q}_h^1 + D\underline{u}}{D}\right)\right] + \\
&\quad \left\{\bar{q}_\alpha^1 \int_{\underline{c}_h}^{b_h^1}\left(\tilde{b} - c_\alpha^1\right)\left[f\left(\sigma\left(\tilde{b}\right)\right)\right]\frac{d\sigma}{d\tilde{b}}d\tilde{b}\right\} G\left(\frac{D\bar{u}_H - \bar{q}_h^1}{D}\right) \qquad (11)
\end{aligned}
$$

where $\tilde{q}_h^1(u) \in \left[0, \bar{q}_h^1\right]$.

From the first order conditions for the optimal bid function $b_h^1(u)$ we obtain:

$$
\begin{aligned}
&\left[\tilde{q}_h^1(u) + \bar{q}_\alpha^1\right]\left\{G\left(\frac{\bar{q}_h^1 + D\bar{u}}{D}\right)\left[1 - F\left(\sigma\left(b_h^1\right)\right)\right] + \right. \\
&\left.\left[1 - G\left(\frac{Du_H - q_h^1}{D}\right)\right]\left[F\left(\sigma\left(b_h^1\right)\right)\right]\right\} + \\
&- \left(b_h^1 - c_h^1\right)\tilde{q}_h^1(u)\,G\left(\frac{\bar{q}_h^1 + D\bar{u}}{D}\right)\left[F'\left(\sigma\left(b_h^1\right)\right)\right] + \\
&+ \left(b_h^1 - c_h^1\right)\tilde{q}_h^1(u)\left[1 - G\left(\frac{Du_H - \bar{q}_h^1}{D}\right)\right]\left[F'\left(\sigma\left(b_h^1\right)\right)\right] + \\
&- \bar{q}_h^1\left(b_h^1 - c_h^1\right)\left[1 - G\left(\frac{\bar{q}_h^1 + D\bar{u}}{D}\right)\right]\left[F'\left(\sigma\left(b_h^1\right)\right)\right] \\
&= 0 \qquad (12)
\end{aligned}
$$

where:

$$\left[F'\left(\sigma\left(b_h^1\right)\right)\right] = f\left(\sigma\left(b_h^1\right)\right)\frac{d\sigma}{db_h^1}$$

In a symmetric Nash equilibrium we must have that $\sigma\left(b_h^1\right) = c_h^1$. Taking this into account and using the fact that:

$$\frac{d\sigma}{db_h^1} = \frac{1}{\frac{db_h^1}{dc_h^1}}$$

we rewrite equation (12) as:

$$\left[\tilde{q}_h^1(u) + \bar{q}_\alpha^1\right] H^M\left(\bar{q}_h^1, c_h^1\right) \frac{db_h^1}{dc_h^1} + \left(b_h^1 - c_h^1\right) \tilde{q}_h^1(u) H'\left(\bar{q}_h^1, c_h^1\right) + \\ - \left(b_h^1 - c_h^1\right) \bar{q}_h^1 \left[1 - G\left(\frac{\bar{q}_h^1 + D\bar{u}}{D}\right)\right] f\left(c_h^1\right) = 0$$

where:

$$H^M\left(\bar{q}_h^1, c_h^1\right) = \left\{G\left(\frac{\bar{q}_h^1 + D\bar{u}}{D}\right)\left[1 - F\left(c_h^1\right)\right] + \left[1 - G\left(\frac{Du_H - q_h^1}{D}\right)\right]\left[F\left(c_h^1\right)\right]\right\}$$

indicates the probability of being the marginal generator and

$$H'\left(\bar{q}_h^1, c_h^1\right) = \frac{\partial H^M(.,.)}{\partial c_h^1}$$

Solving the above differential equation for all values of $c \in \left[c_h^1, \bar{c}_h\right]$, with $b_h^1(\bar{c}_h) = \bar{c}_h$, we obtain:

$$\int_{c_h^1}^{\bar{c}_h} \left[\tilde{q}_h^1(u) + \bar{q}_\alpha^1\right] H^M\left(\bar{q}_h^1, \hat{c}\right) \frac{db_h^1}{d\hat{c}} d\hat{c} + \\ \int_{c_h^1}^{\bar{c}_h} \left(b_h^1 - \hat{c}\right) \tilde{q}_h^1(u) H'\left(\bar{q}_h^1, \hat{c}\right) d\hat{c} + \\ - \int_{c_h^1}^{\bar{c}_h} \left(b_h^1 - \hat{c}\right) \bar{q}_h^1 \left[1 - G\left(\frac{\bar{q}_h^1 + D\bar{u}}{D}\right)\right] f(\hat{c})\, d\hat{c} = 0$$

which can be solved to obtain the following optimal bid function:

$$\begin{aligned} b\left(c_h^1, \tilde{q}_h^1(u)\right) &= \frac{\bar{c}_h\left[\bar{q}_\alpha^1 H^M\left(\bar{q}_h^1, c_h^1\right) - \bar{q}_h^1 H^I\left(\bar{q}_h^1, c_h^1\right)\right]}{H^M\left(\bar{q}_h^1, c_h^1\right)\left[\tilde{q}_h^1(u) + \bar{q}_\alpha^1\right]} + \\ &\quad \frac{\tilde{q}_h^1(u)}{\left[\tilde{q}_h^1(u) + \bar{q}_\alpha^1\right]}\left[c_h^1 + \frac{\int_{c_h^1}^{\bar{c}_h} H^M\left(\bar{q}_h^1, \hat{c}\right) d\hat{c}}{H^M\left(\bar{q}_h^1, c_h^1\right)}\right] + \\ &\quad \frac{\bar{q}_h^1}{\left[\tilde{q}_h^1(u) + \bar{q}_\alpha^1\right]}\left[c_h^1 \frac{H^I\left(\bar{q}_h^1, c_h^1\right)}{H^M\left(\bar{q}_h^1, c_h^1\right)} + \frac{\int_{c_h^1}^{\bar{c}_h} H^I\left(\bar{q}_h^1, \hat{c}\right) d\hat{c}}{H^M\left(\bar{q}_h^1, c_h^1\right)}\right] \end{aligned} \quad (13)$$

where

$$H^I\left(\bar{q}_h^1, c_h^1\right) = \left[1 - F\left(c_h^1\right)\right]\left[1 - G\left(\frac{\bar{q}_h^1 + D\bar{u}}{D}\right)\right]$$

indicates the probability of firm 1 of being inframarginal with plant h.

We first notice from (13) that the bid function is continuous in the range $\left[0, \bar{q}_h^1\right]$ of $\tilde{q}_h^1(u)$, with:

$$
\begin{aligned}
b_h^1\left(c_h^1, \tilde{q}_h^1(u)\right)\Big|_{\tilde{q}_h^1(u)=0} &= \bar{c}_h - \left(\bar{c}_h - c_h^1\right)\frac{\bar{q}_h^1}{\bar{q}_\alpha^1}\left[\frac{H^I\left(\bar{q}_h^1, c_h^1\right)}{H^M\left(\bar{q}_h^1, c_h^1\right)} + \frac{\int_{c_h^1}^{\bar{c}_h} H^I\left(\bar{q}_h^1, \hat{c}\right) d\hat{c}}{H^M\left(\bar{q}_h^1, c_h^1\right)}\right] \\
b_h^1\left(c_h^1, \tilde{q}_h^1(u)\right)\Big|_{\tilde{q}_h^1(u)=\bar{q}_h^1} &= \frac{\bar{c}\left[\bar{q}_\alpha^1 H^M\left(\bar{q}_h^1, c_h^1\right) - \bar{q}_h^1 H^I\left(\bar{q}_h^1, c_h^1\right)\right]}{H^M\left(\bar{q}_h^1, c_h^1\right)\left[\bar{q}_h^1(u) + \bar{q}_\alpha^1\right]} + \\
&\quad + \frac{c_h^1\left[\bar{q}_h^1\left[H^M\left(\bar{q}_h^1, c_h^1\right) + H^I\left(\bar{q}_h^1, c_h^1\right)\right]\right]}{H^M\left(\bar{q}_h^1, c_h^1\right)\left[\bar{q}_h^1(u) + \bar{q}_\alpha^1\right]} + \\
&\quad + \frac{\bar{q}_h^1}{\left[\bar{q}_h^1(u) + \bar{q}_\alpha^1\right]}\frac{\int_{c_h^1}^{\bar{c}_h}\left[H^M\left(\bar{q}_h^1, \hat{c}\right) + H^I\left(\bar{q}_h^1, \hat{c}\right)\right] d\hat{c}}{H^M\left(\bar{q}_h^1, c_h^1\right)}
\end{aligned}
$$

The bid function is increasing with respect to the quantity dispatched when the following condition holds:

$$
\bar{q}_h^1 H^I\left(\bar{q}_h^1, c_h^1\right)\left[\bar{c}_h - c_h^1 - \frac{\int_{c_h^1}^{\bar{c}_h} H^I\left(\bar{q}_h^1, \tilde{c}\right) d\tilde{c}}{H^I\left(\bar{q}_h^1, c_h^1\right)}\right] >
$$

$$
\bar{q}_\alpha^1 H^M\left(\bar{q}_h^1, c_h^1\right)\left[\bar{c}_h - c_h^1 - \frac{\int_{c_h^1}^{\bar{c}_h} H^M\left(\bar{q}_h^1, \tilde{c}\right) d\tilde{c}}{H^M\left(\bar{q}_h^1, c_h^1\right)}\right]
$$

Given the above inequality, general conclusion cannot be established. On the contrary, one can say that the bid function would be increasing with respect to the quantity produced when we set to zero the baseload capacity.

The above considerations indicate that the auction model with random demand and high variability is inefficient even under the assumption of no baseload plants. In particular, the restrictions for a flat bid function with respect to the quantity are either:

$$
\begin{array}{ll}
a) & \bar{c}_h = c_h^1 \\
b) & \bar{q}_\alpha^1 = 0 \text{ and } \left(\tilde{q}_h^1(u) \rightarrow 0\right) \\
c) & \bar{q}_\alpha^1 = 0 \text{ and } \left(\tilde{q}_h^1(u) \rightarrow \bar{q}_h^1\right)
\end{array}
$$

The first rstriction refers to the highest cost bidder, whereas condition $b)$ trivially requires no production at all and finally condition $c)$ requires absence of baseload capacity and full allocation of the peak capacity. In the case $c)$ if we further assume that bidders have symmetric capacities on the (fully dispatched) peak plants, and namely $\bar{q}_h^1 = \bar{q}_h^2 = \bar{q}_h$, then the bid reduces to:

$$b\left(c_h^1, \bar{q}_h^1\right) = c_h^1 + \frac{\int_{c_h^1}^{\bar{c}_h} \left[H^M\left(\bar{q}_h, \hat{c}\right) + H^I\left(\bar{q}_h, \hat{c}\right)\right] d\hat{c}}{H^M\left(\bar{q}_h, c_h^1\right)} - \left[\bar{c}_h - c_h^1\right]\left[\frac{H^I\left(\bar{q}_h, c_h^1\right)}{H^M\left(\bar{q}_h, c_h^1\right)}\right]$$

$$= \frac{\bar{c}_h - G\left(\frac{\bar{q}_h + D\underline{u}}{D}\right) F\left(c_h^1\right)\left[c_h^1 + \frac{\int_{c_h^1}^{\bar{c}_h}(F(\hat{c}))d\hat{c}}{F\left(c_h^1\right)}\right]}{H^M\left(\bar{q}_h, c_h^1\right)}$$

and it is flat with respect to the quantity dispatched.

We conclude that when demand is random and shows a relatively high degree of variability, the power market produces an efficient allocation only in very special situations. In particular, the efficiency of the mechanism is guaranteed only in the presence of an unrealistic degree of symmetry of capacity between bidders. In our model this requires $\bar{q}_h^1 = \bar{q}_h^2$ *and* $\bar{q}_\alpha^1 = \bar{q}_\alpha^2$.

6 Simulations with Italian data

In this section we present the results of a simulation of the operation of the Italian market obtained by using direct observations of cost, capacity and demand levels. The results obtained in the previous sections represent a prediction of different SMP results generated by competitive auctions. These results depend, among other things, on the assumptions made about the degree of concentration of the electricity production sector, on the capacity possessed by generators and on cost levels. The asymmetry of information about costs can be exploited by bidders in different ways according to their potential production strength for each possible realization of the demand for energy and given their costs of generation. If data on costs, capacities and demand were available it would be a non trivial exercise, then, to use these data to construct a sort of benchmark profile of the final allocation of the generation power and the prevailing prices. In turn this might be useful for comparison and evaluation of the theoretical predictions obtained from the above auction models.

6.1 The "full information" merit order

The simulation tries to replicate the functioning of an electricity market[8]. The simulated "full information" merit order is derived as follows. Let:

POW_i = Generation Capacity of plant $i \in (1, ..., 131)$, with

$$POW_{i,Min} \leq POW_i \leq POW_{i,Max}$$

$\overline{POW}_\tau$ = Capacity required for hour $\tau \in (1, ..., 24)$

[8] The model is similar to the procedure introduced in the USA by the PURPA of 1978 (Bushell and Oren, 1994: 6).

$c_{F,i}$ = Fix cost of plant i
$c_{V,i}$ = Variable cost of plant i
The problem the Agency has to solve for each τ is therefore

$$Min \sum_{i=1}^{m_\tau \leq 131} [c_{F,i} + c_{V,i} POW_i]$$

subject to

$$\begin{aligned} POW_{i,Min} &\leq POW_i \leq POW_{i,Max} \\ \sum_{i=1}^{m_\tau \leq 131} POW_i &= \overline{POW}_\tau \end{aligned}$$

where m_τ is the total number of plants dispatched at time (hour) τ. The total amount of generation costs of the $m_\tau \leq 131$ plants dispatched in any hour has to be minimized subject to the constraints given by the capacity limits of each plant and the total power requirement $\overline{POW}_\tau$ announced by the Agency. The solution is obtained by means of the following planning procedure:

1: In the first stage ($t = 0$) of the procedure the 131 plants are sorted in ascending order on the base of variable cost evaluated at POW_{iMin}; then

2: The residual

$$\Delta POW^{(t=0)} = \overline{POW}_\tau - \sum_{i=1}^{131} POW_{i,Min}$$

is calculated;

3: If, as it is likely to be the case, $\Delta POW^{(t=0)} > 0$ and then more power is requested, plants are gradually switched towards the maximum capacity starting from the plant with the lowest cost;

4: Using the lowest cost plant at time 1 of the planning procedure a new residual is computed as

$$\Delta POW^{(t=1)} = \Delta POW^{(t=0)} + [POW_{j,Max} - POW_{j,Min}]$$

where the term in bracket refers to the plant j which is the lowest cost plant;

5: If $\Delta POW^{(t=1)} > 0$ the procedure is repeated using the second lowest cost plant till $\Delta POW^{(t=T)} = 0$ at some T. This determines the value of m_τ and the identity of the marginal plant alongside the identity of all inframarginal plants. At the same time the procedure determines the quantity of power supplied by each dispatched plant.

Given cost observability, the Agency pays to all dispatched plants a price equal to the marginal cost of the plant corresponding to the m_τ position. If this procedure is repeated for the 24 hours of each calendar day during the year the procedure determines the total number of hours each plant is called into operation during that year and a series of 8760 values of the simulated SMPs[9]. The main statistics of this SMP are shown in third row of Table 1. One may notice that the mean SMP is greater than the mean Marginal Cost and that the sign of the skewness of the two distributions is opposite. The majority of the data of the SMP lays above the mean value of the marginal costs since the median of SMP is greater that the mean of the Marginal Cost. The following Fig.1 illustrates the behaviour of the SMP over the entire sample period of 2000.

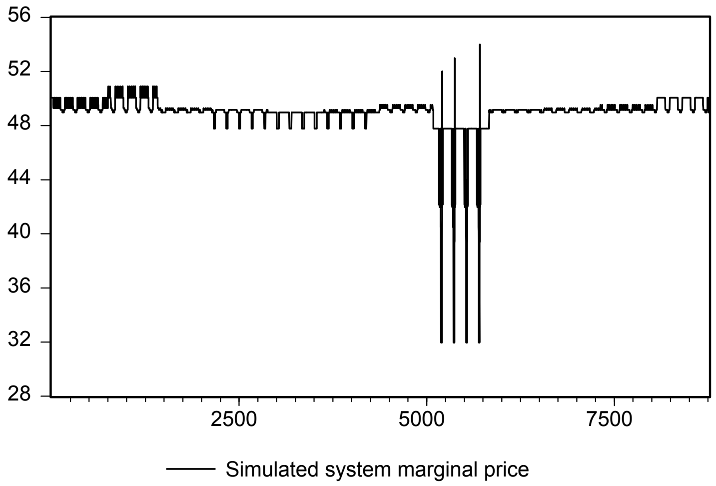

Fig. 1 *The simulated system marginal price*

Considering the results for each plant, we sort in a descending order the units according to the total hours of despatching. Fig. 2 illustrates the above result for year 2000 and Table 1 summarises the main statistics of the data.

[9]In reality given the use of cost information in the planning procedure this should be called the (simulated) system marginal cost.

Table 1 *Summary statistics*

	Obs.	Mean	Median	Max	Min	Std. Dev.
Mg. Cost	131	46.45	45.87	97.42	21.51	15.73
Total Hours	131	3961.3	1496	8760	0.00	3993.9
SMP	8760	48.90	49.17	54.00	31.95	1.63

	Skewness	Kurtosis	Jarque-Bera
Mg. Cost	0.48	3.05	4.99 (0.082)
Total Hours	0.17	1.15	19.42 (0.000)
SMP	-7.13	65.93	1520115 (0.000)

As one can see from Fig. 2 there is a monotonically decreasing (almost everywhere) correspondence between the cost levels and the number of hours of plants' operation (in the simulation, of course). The latter is calculated by summing the hours that each plant was called into operation each hour of each day by the Agency, given the level of the demand that prevailed during each hour of each day. This correspondence defines a merit order in the quantity sense, i.e. it indicates the number of times a plant was the SMP setter during the repetition of the planning procedure. Of the 131 plants only 85 should had been working during the year. A significant level of excess capacity seems to characterize the industry and justify rationing through mechanisms such as auctions.

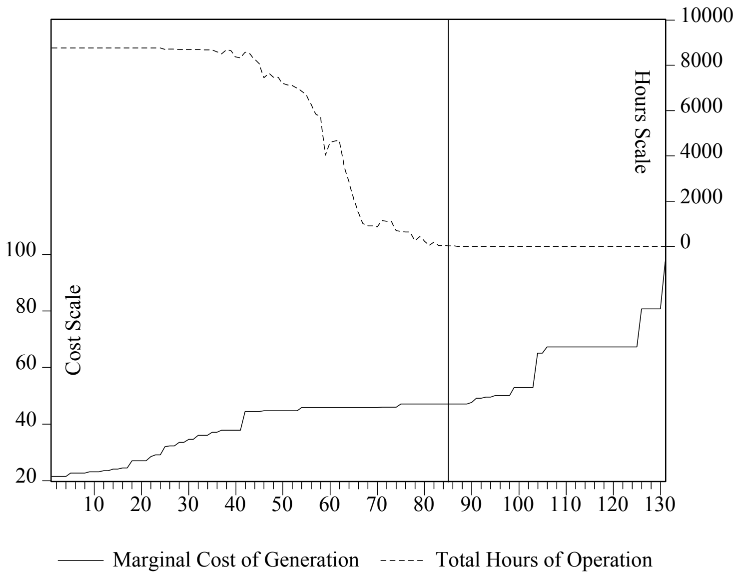

Fig. 2 *Yearly hours of operation and marginal cost of each plant*

Results illustrated in Fig. 2 indicate the operational level in a reference year using data of each production unit. These results however do not fully illustrate the possibility of exercising market power on the part of (multi-plant) firms. To this end we aggregate results at the firm level. The following Table 2 presents a measure of market power of firms defined as the percentage of the times a *firm* has set the SMP during the year. Results are presented for a sample period of ten years (2001-10)[10].

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Enel	85%	72%	61%	93%	63%	16%	16%	17%	23%	20%
Edison	2%	0%	2%	2%	7%	12%	11%	12%	16%	14%
Elettrog	2%	6%	1%	0%	7%	18%	10%	18%	15%	11%
Aem Mi	0%	0%	0%	0%	0%	1%	4%	4%	3%	1%
Acea	0%	0%	0%	1%	1%	2%	0%	0%	0%	0%
Eni Power	0%	0%	0%	0%	1%	11%	14%	14%	10%	17%
Aem/Asm	0%	3%	5%	0%	1%	0%	0%	0%	0%	0%
Eurogen	11%	20%	31%	3%	9%	16%	9%	14%	7%	13%
Caffaro	0%	0%	0%	0%	3%	7%	5%	0%	0%	1%
Interpower	0%	0%	0%	0%	4%	5%	9%	4%	6%	5%
CentroE	0%	0%	0%	0%	1%	4%	6%	3%	4%	5%
Tracteb	0%	0%	0%	0%	3%	2%	5%	2%	5%	5%
Sondel	0%	0%	0%	0%	0%	5%	11%	10%	11%	8%
Sogetel	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Aem To	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
South	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Tab. 2 *Degree of market power*

As one can see markets structure in this benchmark setting evolves over time and shows that the high degree of market power possessed by ENEL (the former public monopoly) at the beginning of the sample period should be expected to be significantly reduced by 2010. The situation prevailing at the start of the sample period corresponds to the industry structure we considered in the model we first analysed (a large producer vs. a fringe). As time passes by the structure of the industry assumes the characters of an oligopolistic setting.

7 Conclusions

In this paper we analyzed some possible outcomes of the competition in the power market regulated by a competitive multi-unit auction. Contrary to some part of the literature we assumed cost uncertainty on the part of multi-plant bidders. We examined as well cases with demand uncertainty. We tried and characterized the industry structure in order to resemble as much as we could the likely profile of the Italian wholesale electricity market which is expected to start its operation soon.

We first analyzed a model where a large firm competes with a fringe of small bidders in a case of both fixed and random demand. We found that

[10] This forecast is made on the basis of a dataset containing information about the predictions on the time evolution of the most relevant variables of the industry. In particular, the dataset contains data on the evolution of the demand of electricity as well as costs, capacity of operating plants and tecnology. This dataset is proprietary of REF-IRS.

the incentive to bid above costs increases with the baseload capacity whereas no general conclusion can be drawn with respect to peak quantity. In the limit case in which all peak capacity is sold in equilibrium we derived that the conditions for the optimal bid made by the marginal plant imply that the markup over costs are equal to the expected difference between the cost of this plant and the cost of the generator coming next in the merit order. Results extend to the case of uncertain demand.

The second model we analyzed is a duopoly competition. In this case we derived a symmetric equilibrium bid function. The optimal bid turns out to be a weighted average of the highest cost and that bid that would have been optimal in a discriminative auction. The latter is given by the expected value of the cost of the next firm in the merit order. The weights are given by the proportion of baseload and peak quantity over total quantity supplied and are such that the larger the capacity of the baseload plant, the higher the weight associated to the cost ceiling. This finding extends Brunekreeft (2001) result to the case of cost uncertainty. In the case of duopoly with demand uncertainty results are not so clear cut. In particular, with low variability of demand we obtained a result similar to the one just discussed whereas with high variability of demand general conclusions cannot be established. We have shown that there exists an equilibrium bidding function monotonically increasing with respect to private costs but the bid function is not flat with respect to the quantity sold by the peak plant. This means that there is ex-post inefficiency in the allocation produced by the competitive mechanism. Efficiency of the mechanism holds only in special cases which require an high level of symmetry between bidders (same peak and baseload capacities) and the fact that full capacity is sold in equilibrium. This result parallels *Lemma 1* of Ausubel and Cramton (1998).

Although the above conclusions might look like ambiguous, still some policy implications can be drawn on the basis of the above models. For instance we can emphasize the role of base capacity. Base capacity only increases the bid shading which implies that an high level of total capacity induces higher energy prices with obvious reduction in consumer's surplus. In terms of productive efficiency the above result implies that the same level of operational efficiency achievable by integrated electric utility cannot be maintained in a system where generation is acquired through a competitive auction. There is a trade-off between the advantages brought about by competitive bids and the disadvantages represented by the possible forsaking of scale/scope efficiency associated to regulatory decision of breaking up existing large firms. In the lights of our results, the above trade-off might bend towards the latter corner.

In any case, the relevancy of the multi-unit aspect of the auction for the efficiency of the allocation introduces an interesting question in the public policy towards the sector. How should a quasi-monopolist such as ENEL be broken up? This question is empirically relevant given the ongoing process of selling the three Gencos disentangled from ENEL. Our results suggest that policy measures should determine the conditions most favorable to the emergence of basically equal firms, i.e. firms endowed with a similar capacity mix.

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